Green Dyadics in Composite Chiral-Ferrite Medium by Cylindrical Vector Wave Functions

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Abstract. — Composite chiral-ferrite medium, which is a generalization of the well-studied chiral medium, has potential application in chirality management In the present investigation, based on the concept of spectral eigenwaves, eigenfunction expansion of the Green dyadics in an unbounded composite chiral-ferrite medium is developed in the forms of the cylindrical vector wave functions. The formulations are considerably simplified by analytically evaluating the integrals with respect to the spectral longitudinal and radial wavenumbers, respectively. The analysis indicates that the solutions of the source-incorporated Maxwell's equations for a homogeneous composite chiral-ferrite medium, which can be represented in sum-integral forms of the cylindrical vector wave functions, are composed of two (or four) eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave The Green dyadics of planarly and cylindrically multilayered structures consisting of composite chiral-ferrite media can be straightforwardly obtained by applying the method of scattering superposition and appropriate boundary conditions, respectively. The present formulations, which can be theoretically verified by comparing their special forms with the already existed results, provide fundamental basis to analyze the physical phenomena of the composite chiral-ferrite media.

1. Introduction

The concept of vector wave functions was first proposed by Hansen [1] to solve the source-free Maxwell's equations in isotropic media This vector-wave-function approach has been intensively developed by Felsen and Marcuvitz [2], Morse and Feshbach [3], and Tai [4], to investigate the source-incorporated electromagnetic boundary value phenomena of isotropic media. It has been discovered that for some types of electromagnetic boundary value problems of isotropic media (*e.g.*, microstrip wraparound antennas [5], circular-shaped microwave radiators [6,7], and excitations of cylindrical waveguides and cavities [8]), field representations and Green dyadics by the cylindrical vector wave functions are more useful than those by the planar vector wave functions. Recently, field representations by the cylindrical vector wave functions of isotropic

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media were presented for the source-free gyroelectric chiral media [9], composite chiral-ferrite media [10], reciprocal uniaxial bianisotropic media [11], and uniaxial bianisotropic-ferrite media [12]. However, analytical solutions to the source-incorporated Maxwell's equations in any given complex media still need to be developed, so as to provide methodological convenience in studying the physical phenomena of these materials.

The Green dyadic is one of the basic tools that are used to solve the source-incorporated Maxwell's equations. It is useful both in analyzing radiation problems [4,13] and in constructing integral equations for scattering phenomena [14, 15]. The general representation of the Green dyadic expressed in terms of an expansion of the vector wave functions are required to study Raman and fluorescent scattering by active molecules embedded in a particle [16, 17], as well as to establish T-matrix formulation from Huygen's principle and extinction theorem [18, 19]. Furthermore, eigenfunction expansion of the Green dyadics could also provide fundamental insight into the physical process of the material under consideration. However, much effort is still required in order to obtain the Green dyadics in any given complex media when expressed in the full eigenfunction expansion of the vector wave functions.

With recent advances in polymer synthesis techniques, increasing attention is being attracted to the analysis of interaction between electromagnetic waves and chiral media [11, 20, 21], in order to determine how to use these materials to provide better solutions to current engineering problems It has been discovered chiral materials can be utilized to construct anti-reflection coatings, novel reciprocal microwave components, and antenna radomes, whose physical behaviours are dominated by the chirality degree [11,20,21]. However, only limited methods exist in the control of chirality degree once chiral materials are fabricated, except by introducing certain forms of anisotropy. With chirality management as investigation motivation, Engheta *et al.* [22] investigated the propagation characteristics of electromagnetic waves in unbounded Faraday chiral media, which blend the effects of Faraday rotation with those of optical activity. The general formulations of composite chiral-ferrite media, such as nonreciprocal properties, dyadic Green's functions in unbounded space, dispersion relations and polarization characteristics, were developed by Krowne [23] Most recently, solutions of source-free Maxwell's equations for a homogeneous composite chiral-ferrite medium are represented in terms of the cylindrical vector wave functions [10]. Nevertheless, much effort is still needed in order to achieve a throughout understanding of the chirality management.

From a phenomenological point of view, a homogeneous composite chiral-ferrite medium can be characterized by the set of constitutive relations [10, 23]

$$\mathbf{D} = \varepsilon\mathbf{E} + i\xi_c\mathbf{H}, \tag{1a}$$

$$\mathbf{B} = \bar{\mu}\cdot\mathbf{H} - i\xi_c\mathbf{E} \tag{1b}$$

where $\bar{\mu} = \mu_t\bar{\mathbf{I}}_t + \mu_z\mathbf{e}_z\mathbf{e}_z - ig\mathbf{e}_z \times \bar{\mathbf{I}}_t$ is the modified permeability dyadic taking into account of the contributions due to chirality. ξ_c and ε are the chirality parameter and permittivity, respectively. Here, $\bar{\mathbf{I}}_t = \mathbf{e}_x\mathbf{e}_x + \mathbf{e}_y\mathbf{e}_y$ denotes the transverse unit dyadic, and $\mathbf{e}_j$ represents the unit vector in the j direction. Instead of three parameters for the well-studied chiral media [24, 25], we are facing a medium with five scalar parameters. It is apparent the constitutive dyadics of the medium satisfy the nonreciprocal conditions [26] as well as the uniformity constraint condition [27]. For a lossless composite chiral-ferrite medium, the constitutive parameters ε, μ_t, μ_z, g and ξ_c are all real, which are assumed throughout the present investigation.

A composite chiral-ferrite medium, formed by immersing chiral objects in a magnetized ferrite, is a subset of the wider class referred to as bianisotropic media. Excellent work on general bianisotropic media has been reported by Post [28], Kong [26], and Chen [29] among others In contradistinction to these general considerations, the present contribution is intended to develop the eigenfunction expansion of the Green dyadics in composite chiral-ferrite medium in terms of the cylindrical vector wave functions. Based on the concept of spectral eigenwaves, the formulations are considerably simplified by analytically evaluating the integrals with respect to the spectral longitudinal and radial wavenumbers, respectively This extended method, which is standard and straightforward, leads to two sets of the eigenfunction expansion of the Green dyadics in terms of the cylindrical vector wave functions. The analysis indicates that the solutions of the source-incorporated Maxwell's equations for a homogeneous composite chiral-ferrite medium, which can be represented in sum-integral forms of the cylindrical vector wave functions, are composed of two (or four) eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. It is also found that the Sommerfeld-Weyl-type integrals of dipole radiation in a composite chiral-ferrite medium involve only those Sommerfeld-Weyl-type integrals of dipole radiation in an isotropic medium The present formulations can be used to construct the Green dyadics of planarly and cylindrically multilayered structures consisting of composite chiral-ferrite media, by employing the method of scattering superposition [4,24,25] and appropriate electromagnetic boundary conditions, respectively. The greatest advantage of the Green dyadics, which are represented in the forms of the eigenfunction expansion, is that they provide fundamental insight into the physical process of the composite chiral-ferrite medium, and lay the theoretical foundation to study the source-incorporated electromagnetic phenomena involving composite chiral-ferrite media (*e.g.*, Raman and fluorescent scattering by active molecules embedded in a composite chiral-ferrite medium).

In the following analysis, the harmonic $\exp(i\omega t)$ time dependence is assumed and suppressed throughout.

2. Eigenwaves in Composite Chiral-Ferrite Medium

Substituting the constitutive relations (1a) and (1b) into the source-incorporated Maxwell's equations, a compact form of the field equations in the composite chiral medium is obtained:

$$\begin{pmatrix} \omega\varepsilon\bar{\mathbf{I}} & i\omega\xi_c\bar{\mathbf{I}} + i\nabla\times \\ -i\omega\xi_c\bar{\mathbf{I}} - i\nabla\times & \omega\bar{\mu} \end{pmatrix} \begin{pmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} i\mathbf{J}(\mathbf{r}) \\ i\mathbf{M}(\mathbf{r}) \end{pmatrix}, \tag{2}$$

where $\bar{\mathbf{I}}$ denotes 3×3 unit dyadic, $\mathbf{J}$ and $\mathbf{M}$ represent the electric and magnetic exciting currents, respectively

To examine the physical properties of the eigenwaves, Fourier transformation for the electromagnetic fields and exciting sources is introduced

$$\mathbf{F}(\mathbf{r}) = \frac{1}{8\pi^3}\int_{-\infty}^{\infty} \mathbf{F}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}, \tag{3}$$

where $\mathbf{F} = \mathbf{E}$, $\mathbf{H}$, $\mathbf{J}$ or $\mathbf{M}$, $\mathbf{k} = k_x\mathbf{e}_x + k_y\mathbf{e}_y + k_z\mathbf{e}_z$. Then, equation (2) can be rewritten in the Fourier spectral domain

$$\begin{pmatrix} \omega\varepsilon\bar{\mathbf{I}} & i\omega\xi_c\bar{\mathbf{I}} + \mathbf{k}\times \\ -i\omega\xi_c\bar{\mathbf{I}} - \mathbf{k}\times & \omega\bar{\mu} \end{pmatrix} \begin{pmatrix} \mathbf{E}(\mathbf{k}) \\ \mathbf{H}(\mathbf{k}) \end{pmatrix} = \begin{pmatrix} i\mathbf{J}(\mathbf{k}) \\ i\mathbf{M}(\mathbf{k}) \end{pmatrix}. \tag{4}$$

For the sake of brevity, (4) is denoted as

$$\bar{\mathbf{L}}\cdot\boldsymbol{\Psi}(k) = \boldsymbol{\Phi}(k), \tag{5}$$

where $\bar{\mathbf{L}}$ is a Hermitian operator (*i.e.*, $\bar{\mathbf{L}} = \bar{\mathbf{L}}^{*\mathrm{T}}$, where the superscripts asterisk and T denote complex conjugate and transpose, respectively). Here, $\boldsymbol{\Psi}(\mathbf{k}) = [\mathbf{E}(\mathbf{k}),\ \mathbf{H}(\mathbf{k})]^{\mathrm{T}}$, and $\boldsymbol{\Phi}(\mathbf{k}) = [i\mathbf{J}(k), i\mathbf{M}(\mathbf{k})]^{\mathrm{T}}$.

The characteristic equation, which determines the wavenumbers of the eigenwaves propagating in the composite chiral-ferrite medium, can be straightforwardly obtained by requiring the determinant of operator $\bar{\mathbf{L}}$ be zero. Algebraic manipulation results in

$$ak_\rho^4 + [(k_z^2 - a)(a + a') + c^2 - ab^2]k_\rho^2 + [(k_z^2 - a)^2 - (bik_z - c)^2]a' = 0 \tag{6}$$

where $k_\rho = (k_x^2 + k_y^2)^{1/2}$, and

$$\begin{aligned} a &= \omega^2 \varepsilon \mu_\mathrm{t} \left[1 - \frac{\xi_\mathrm{c}^2}{\varepsilon \mu_\mathrm{t}}\right], \\ b &= -2i\xi_\mathrm{c}, \\ c &= \omega^2 \varepsilon g, \\ a' &= \omega^2 \varepsilon \mu_z \left[1 - \frac{\xi_\mathrm{c}^2}{\varepsilon \mu_z}\right]. \end{aligned} \tag{7}$$

It is obvious that the characteristic equation (6) is an even function of k_ρ We can regard this characteristic equation (6) as a function of k_ρ (or k_z), where k_ρ (or k_z) is determined by k_z (or k_ρ). The roots of (6) are designated as $k_\rho = k_{\rho q}$ (or $k_z = k_{zq}$), where $q = 1$, 2, 3 and 4. It is worthy to note the important property of the roots of (6): $k_{\rho q}$ (or k_{zq}) is independent of ϕ_k, with $\phi_k = \tan^{-1}(k_y/k_x)$.

The eigenwave corresponding to the q-th root of (6), expressed in a circular cylindrical coordinate system, can be derived by substituting $k_\rho = k_{\rho q}$ or $k_z = k_{zq}$ in the following expression

$$\mathbf{\Psi}_q^\sigma(\mathbf{k}) = \begin{pmatrix} E_{q\rho}^\sigma(\mathbf{k}) \\ E_{q\phi}^\sigma(\mathbf{k}) \\ E_{qz}^\sigma(\mathbf{k}) \\ H_{q\rho}^\sigma(\mathbf{k}) \\ H_{q\phi}^\sigma(\mathbf{k}) \\ H_{qz}^\sigma(\mathbf{k}) \end{pmatrix} = \begin{pmatrix} C_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) + D_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) \\ -C_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) + D_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) \\ -\frac{1}{\omega\varepsilon}[i\omega\xi_\mathrm{c} + k_\rho B_q^\sigma(k_\rho, k_z)] \\ A_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) + B_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) \\ -A_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) + B_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) \\ 1 \end{pmatrix}, \tag{8}$$

with $\phi = \tan^{-1}(y/x)$, $\sigma = \rho$ for $k_\rho = k_{\rho q}$, and $\sigma = z$ for $k_z = k_{zq}$ Here, the spectral parameters are found to be

$$A_q^\sigma(k_\rho, k_z) = \frac{k_\rho[k_z(k_\rho^2 + k_z^2 - a + b^2) + ibc]}{E_q^\sigma(k_\rho, k_z)}, \tag{9}$$

$$B_q^\sigma(k_\rho, k_z) = \frac{ik_\rho(iab - ck_z)}{E_q^\sigma(k_\rho, k_z)}, \tag{10}$$

$$E_q^\sigma(k_\rho, k_z) = (k_z^2 - a)(k_\rho^2 + k_z^2 - a) - (ibk_z - c)^2, \tag{11}$$

$$C_q^\sigma(k_\rho, k_z) = \frac{1}{\omega\varepsilon}[-i\omega\xi_\mathrm{c} A_q^\sigma(k_\rho, k_z) + k_z B_q^\sigma(k_\rho, k_z)], \tag{12}$$

and

$$D_q^\sigma(k_\rho, k_z) = \frac{1}{\omega\varepsilon}[-i\omega\xi_\mathrm{c} B_q^\sigma(k_\rho, k_z) + k_\rho - k_z A_q^\sigma(k_\rho, k_z)]. \tag{13}$$

To reveal the bi-orthogonality property of these eigenwaves, equation (5) should be rewritten in other forms First, regarding $k_{\rho q}$ as the roots of the characteristic equation (6), (5) is rewritten as

$$\bar{\mathbf{A}}_1 \cdot \mathbf{\Psi}_q^\rho(\mathbf{k}) - k_{\rho q}\bar{\mathbf{B}}_1 \cdot \mathbf{\Psi}_q^\rho(\mathbf{k}) = \mathbf{\Phi}(\mathbf{k}), \tag{14}$$

where both $\bar{\mathbf{A}}_1$ and $\bar{\mathbf{B}}_1$ are Hermitian operators. These eigenwaves $\mathbf{\Psi}_q^\rho(\mathbf{k})$, which form a complete set in the spectral space [29], are bi-orthogonality [29,30]: $\mathbf{\Psi}_p^{\rho*}(\mathbf{k})\cdot\bar{\mathbf{B}}_1\cdot\mathbf{\Psi}_q^\rho(\mathbf{k}) = N_p^2\delta_{pq}$. Here, δ_{pq} denotes the Kronecker delta function (*i.e.*, it is 1 for $p = q$, and 0 for $p \neq q$).
An alternative useful rewritten form of (5) is

$$\bar{\mathbf{A}}_2 \cdot \mathbf{\Psi}_q^z(\mathbf{k}) - k_{zq}\bar{\mathbf{B}}_2 \cdot \mathbf{\Psi}_q^z(\mathbf{k}) = \mathbf{\Phi}(\mathbf{k}), \tag{15}$$

where both $\bar{\mathbf{A}}_2$ and $\bar{\mathbf{B}}_2$ are Hermitian operators, and the roots of the characteristic equation (6) are considered to be k_{zq}. The eigenwaves of (15), which form a complete set in the spectral space [29], are also bi-orthogonality [29, 30]: $\mathbf{\Psi}_p^{\rho*}(\mathbf{k}) \cdot \bar{\mathbf{B}}_2 \cdot \mathbf{\Psi}_q^z(\mathbf{k}) = \mathrm{M}_p^2\delta_{pq}$.
Based on the completeness properties of the above-presented eigenwaves $\mathbf{\Psi}_q^z(\mathbf{k})$ and $\mathbf{\Psi}_q^\rho(\mathbf{k})$ the solutions of the spectral source-incorporated equation (5) can be represented in terms of these eigenwaves [2, 29, 30]:

$$\mathbf{\Psi}(\mathbf{k}) = \sum_q \frac{\mathbf{\Psi}_q^z(\mathbf{k})\mathbf{\Psi}_q^{z*}(\mathbf{k})}{(k_{zq} - k_z)M_q^2} \cdot \mathbf{\Phi}(\mathbf{k}), \tag{16}$$

or

$$\mathbf{\Psi}(\mathbf{k}) = \sum_q \frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho*}(\mathbf{k})}{(k_{\rho q} - k_\rho)N_q^2} \cdot \mathbf{\Phi}(\mathbf{k}). \tag{17}$$

In this way, the solutions of the spectral source-incorporated Maxwell's equation (4) are represented in terms of the spectral eigenwaves in the composite chiral-ferrite medium. These expressions, (16) and (17), are our starting point in constructing the eigenfunction expansion of the Green dyadics, as will be reported in detail in the following analysis.

3. Green Dyadics in Composite Chiral-Ferrite Medium

For the sake of simplicity, we define the Green dyadics $\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$ in the composite chiral-ferrite medium as

$$\begin{pmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{pmatrix} = \int_{V'} \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot \begin{pmatrix} i\mathbf{J}(\mathbf{r}') \\ i\mathbf{M}(\mathbf{r}') \end{pmatrix} \mathrm{d}V', \tag{18}$$

where V' is the volume occupied by the electric and magnetic exciting currents. The definition (18) indicates that the electromagnetic fields associated with the current sources can be expressed as a convolution of the current distribution and the three-dimensional free-space Green dyadics.

Using the definition of Green dyadics (18) and equations (16) and (17), the Green dyadics in the composite chiral-ferrite medium can be represented in terms of the spectral eigenwaves

$$\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \mathrm{d}\mathbf{k} \sum_q \frac{\mathbf{\Psi}_q^z(\mathbf{k})\mathbf{\Psi}_q^{z*}(\mathbf{k})}{(k_{zq} - k_z)M_q^2} \mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}, \tag{19}$$

$$\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \mathrm{d}\mathbf{k} \sum_q \frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho*}(\mathbf{k})}{(k_{\rho q} - k_\rho)N_q^2} \mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}. \tag{20}$$

Here, the convolution theorem of Fourier transformation has been employed.

It is helpful to mention that equation (19) is suitable to construct the Green dyadics of planarly multilayered composite chiral-ferrite media, while (20) is a useful tool to formulate the Green dyadics of a cylindrically multilayered structure consisting of composite chiral-ferrite media.

To represent the Green dyadics in the forms of the eigenfunction expansion in terms of the cylindrical vector wave functions, integrals with respect to the spectral longitudinal and radial wavenumbers in equations (19) and (20) respectively will be evaluated analytically

3.1. Analytical Evaluation of the Integral with Respect to the Spectral Longitudinal Wavenumber. — In this subsection, we will try to represent equation (19) in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions. For this purpose, the integral with respect to the spectral longitudinal wavenumber k_z is analytically evaluated by using the residue method, which results in

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \begin{pmatrix} \bar{\mathbf{G}}_{\rm ee}(\mathbf{r},\mathbf{r}') & \bar{\mathbf{G}}_{\rm em}(\mathbf{r},\mathbf{r}') \\ \bar{\mathbf{G}}_{\rm me}(\mathbf{r},\mathbf{r}') & \bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}') \end{pmatrix} \tag{21}$$
$$= \frac{i}{8\pi^2}\int_0^\infty dk_\rho \int_{\phi_k=0}^{2\pi} d\phi_k \sum_{q=1}^{4} \frac{\mathbf{\Psi}_q^z(\mathbf{k})\mathbf{\Psi}_q^{z*}(\mathbf{k})}{M_q^2} e^{-ik_{zq}(z-z')} e^{-ik_\rho\rho\cos(\phi-\phi_k)} e^{ik_\rho\rho'\cos(\phi'-\phi_k)},$$

with $\rho = (x^2+y^2)^{1/2}$. Here, the 3×3 dyadics $\bar{\mathbf{G}}_{\rm ee}(\mathbf{r},\mathbf{r}')$ and $\bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}')$ are the Green dyadics of electric and magnetic types, respectively; while $\bar{\mathbf{G}}_{\rm em}(\mathbf{r},\mathbf{r}')$ and $\bar{\mathbf{G}}_{\rm me}(\mathbf{r},\mathbf{r}')$ are the pseudo-type Green dyadics. It should be recognized that the following formulations are essentially based on the fact that the spectral longitudinal wavenumber k_{zq} is independent of the spectral azimuthal angle ϕ_k.

Substituting into (21) the explicit expression of $\mathbf{\Psi}_q^z(\mathbf{k})$ and the well-known identities

$$e^{-ik_\rho\rho\cos(\phi-\phi_k)} = \sum_{n=-\infty}^{\infty} (-i)^n J_n(k_\rho\rho) e^{-in(\phi-\phi_k)}, \tag{22}$$
$$e^{ik_\rho\rho'\cos(\phi'-\phi_k)} = \sum_{m=-\infty}^{\infty} (i)^m J_m(k_\rho\rho') e^{im(\phi'-\phi_k)}, \tag{23}$$

after cumbersome mathematical manipulation by grouping properly the terms involving the integral for the ϕ_k variable and introducing the cylindrical vector wave functions, we end up with

$$\begin{aligned}\bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}') &= \frac{i}{8\pi}\int_0^\infty dk_\rho \sum_{q=1}^{4}\frac{1}{M_q^2}\sum_{n=-\infty}^{\infty}(-1)^n \\ &\times[a_q^z(k_\rho,k_{zq})\mathbf{M}_n^{(1)}(k_\rho,k_{zq}) + b_q^z(k_\rho,k_{zq})\mathbf{N}_n^{(1)}(k_\rho,k_{zq}) \\ &+c_q^z(k_\rho,k_{zq})\mathbf{L}_n^{(1)}(k_\rho,k_{zq})][a_q^{z'}(k_\rho,k_{zq})\mathbf{M}_{-n}^{(1)'}(k_\rho,-k_{zq}) \\ &+b_q^z(k_\rho,k_{zq})\mathbf{N}_{-n}^{(1)'}(k_\rho,-k_{zq}) + c_q^{z'}(k_\rho,k_{zq})\mathbf{L}_{-n}^{(1)'}(k_\rho,-k_{zq})],\end{aligned} \tag{24}$$

where the primes over the vector wave functions denote that they are evaluated at $\mathbf{r}'$. Here, the techniques of mathematical manipulation are similar to those we have used in [9–12] to obtain the field representations in the source-free regions. The expansion coefficients are found to be

$$a_q^\sigma(k_\rho,k_z) = -\frac{2iB_q^\sigma(k_\rho,k_z)}{k_\rho}, \tag{25}$$
$$b_q^\sigma(k_\rho,k_z) = -\frac{2k_q A_q^\sigma(k_\rho,k_z)}{k_\rho k_z} + \frac{2}{k_q}\left[1+\frac{k_\rho A_q^\sigma(k_\rho,k_z)}{k_z}\right], \tag{26}$$
$$c_q^\sigma(k_\rho,k_z) = \frac{2ik_z}{k_q^2}\left[1+\frac{k_\rho A_q^\sigma(k_\rho,k_z)}{k_z}\right], \tag{27}$$

with $k_q = (k_z^2+k_\rho^2)^{1/2}$, $\sigma = z$ and $k_z = k_{zq}$. $a_q^{\sigma'}(k_\rho,k_z)$, $b_q^{\sigma'}(k_\rho,k_z)$ and $c_q^{\sigma'}(k_\rho,k_z)$ can separately be derived from $a_q^\sigma(k_\rho,k_z)$, $b_q^\sigma(k_\rho,k_z)$ and $c_q^\sigma(k_\rho,k_z)$, with the replacement of $A_q^\sigma(k_\rho,k_z)$

and $B_q^\sigma(k_\rho, k_z)$ by their complex conjugates, respectively. The roots $k_z = k_{zq}$ of the characteristic equation (6) are chosen such that $\Re(k_{zq}) > 0$ for $z > z'$, and $\Re(k_{zq}) < 0$ for $z < z'$, where $\Re$ denotes the real part of a complex function. The cylindrical vector wave functions are defined as

$$\mathbf{M}_n^{(j)}(k_\rho, k_z) = \nabla \times [\Psi_n^{(j)}(k_\rho, k_z)\mathbf{e}_z], \tag{28}$$

$$\mathbf{N}_n^{(j)}(k_\rho, k_z) = \frac{1}{k_q}\nabla \times \mathbf{M}_n^{(j)}(k_\rho, k_z), \tag{29}$$

$$\mathbf{L}_n^{(j)}(k_\rho, k_z) = \nabla\Psi_n^{(j)}(k_\rho, k_z), \tag{30}$$

where the generating function is

$$\Psi_n^{(j)}(k_\rho, k_z) = Z_n^{(j)}(k_\rho \rho)\mathrm{e}^{-i(k_z z + n\phi)}, \tag{31}$$

with

$$Z_n^{(j)}(k_\rho, \rho) = \begin{cases} J_n(k_\rho, \rho) & j = 1 \\ Y_n(k_\rho, \rho) & j = 2 \\ H_n^{(1)}(k_\rho, \rho) & j = 3 \\ H_n^{(2)}(k_\rho, \rho) & j = 4. \end{cases} \tag{32}$$

The Green dyadic of electric type $\bar{\mathbf{G}}_{\mathrm{ee}}(\mathbf{r}, \mathbf{r}')$ can be obtained from $\bar{\mathbf{G}}_{\mathrm{mm}}(\mathbf{r}, \mathbf{r}')$ with the replacement of a_q^z, b_q^z, c_q^z, $a_q^{z'}$, $b_q^{z'}$, $c_q^{z'}$ by d_q^z, e_q^z, f_q^z, $d_q^{z'}$, $e_q^{z'}$, $f_q^{z'}$, respectively. Here, the expansion coefficients are determined as

$$d_q^\sigma(k_\rho, k_z) = -\frac{2iD_q^\sigma(k_\rho, k_z)}{k_\rho}, \tag{33}$$

$$e_q^\sigma(k_\rho, k_z) = -\frac{2k_z C_q^\sigma(k_\rho, k_z)}{k_\rho k_q} - \frac{2[i\omega\xi_{\mathrm{c}} + k_\rho B_q^\sigma(k_\rho, k_z)]}{k_q \omega\varepsilon}, \tag{34}$$

$$f_q^\sigma(k_\rho, k_z) = \frac{2i}{k_q^2}\left[k_\rho C_q^\sigma(k_\rho, k_z) - \frac{k_z[i\omega\varepsilon_{\mathrm{c}} + k_\rho B_q^\sigma(k_\rho, k_z)]}{\omega\varepsilon}\right], \tag{35}$$

for $\sigma = z$ and $k_z = k_{zq}$ $d_q^{\sigma'}(k_\rho, k_z)$, $e_q^{\sigma'}(k_\rho, k_z)$ and $f_q^{\sigma'}(k_\rho, k_z)$ can separately be obtained from $d_q^\sigma(k_\rho, k_z)$, $e_q^\sigma(k_\rho, k_z)$ and $f_q^\sigma(k_\rho, k_z)$, with the replacement of $C_q^\sigma(k_\rho, k_z)$, $D_q^\sigma(k_\rho, k_z)$ and $[i\omega\xi_{\mathrm{c}} + k_\rho B_q^\sigma(k_\rho, k_z)]$ by their complex conjugates, respectively. The pseudo-type Green dyadics $\bar{\mathbf{G}}_{\mathrm{em}}(\mathbf{r}, \mathbf{r}')$ can be obtained from $\bar{\mathbf{G}}_{\mathrm{me}}(\mathbf{r}, \mathbf{r}')$ with the substitution of a_q^z, b_q^z, c_q^z by d_q^z, e_q^z, f_q^z, respectively; and $\bar{\mathbf{G}}_{\mathrm{me}}(\mathbf{r}, \mathbf{r}')$ can be derived from $\bar{\mathbf{G}}_{\mathrm{mm}}(\mathbf{r}, \mathbf{r}')$, with the replacement of $a_q^{z'}$, $b_q^{z'}$, $c_q^{z'}$ by $d_q^{z'}$, $e_q^{z'}$, $f_q^{z'}$ separately.

It should be mentioned that the present eigenfunction expansion of the Green dyadics can be reduced to the counterparts of a reciprocal chiral medium [24], if letting $\mu_{\mathrm{t}} = \mu_z = \mu$ and $g = 0$ in the constitutive relations. This set of the eigenfunction representation of the Green dyadics can be used to construct the Green dyadics of planarly multilayered composite chiral-ferrite media, by applying the method of scattering superposition [4,24] and appropriate electromagnetic boundary conditions.

Straightforward mathematical analysis reveals that for dipole sources parallel to the z-axis, only the terms corresponding to $n = 0$ exist for the Green dyadics, while the Green dyadics of dipole sources perpendicular to the z-axis contain only the $n = 1$ terms. Therefore, Sommerfeld-Weyl-type integrals of dipole radiation in a composite chiral-ferrite medium involve only those Sommerfeld-Weyl-type integrals of dipole radiation in an isotropic medium [31]. So, various approximate, asymptotic, and numerical method for Sommerfeld-Weyl-type integrals [31] can be applied to study the electromagnetic resonance, radiation, propagation, and scattering phenomena of planarly multilayered composite chiral-ferrite media.

3.2. Analytical Evaluation of the Integral with Respect to the Spectral Radial Wavenumber. — In this subsection, we will try to represent equation (20) in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions. To this end, employing the identities (22) and (23), the integral with respect to the spectral radial wavenumber k_ρ is analytically evaluated by applying the residue calculus through a modified contour in the k_ρ plane, which results in

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \begin{pmatrix} \bar{\mathbf{G}}_{\rm ee}(\mathbf{r},\mathbf{r}') & \bar{\mathbf{G}}_{\rm em}(\mathbf{r},\mathbf{r}') \\ \bar{\mathbf{G}}_{\rm me}(\mathbf{r},\mathbf{r}') & \bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}') \end{pmatrix}$$

$$= \begin{cases} \dfrac{i}{16\pi^2}\displaystyle\int_{-\infty}^{\infty} dk_z \int_{\phi_k=0}^{2\pi} d\phi_k \sum_q \frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho *}(\mathbf{k})}{N_q^2} e^{-ik_z(z-z')} \sum_{n=-\infty}^{\infty} (-i)^n J_n(k_{\rho q}\rho) \\ \qquad \times e^{-in(\phi-\phi_k)} \displaystyle\sum_{m=-\infty}^{\infty} i^m H_m^{(2)}(k_{\rho q}\rho') e^{im(\phi'-\phi_k)}, \qquad \rho \le \rho'. \\ \dfrac{i}{16\pi^2}\displaystyle\int_{-\infty}^{\infty} dk_z \int_{\phi_k=0}^{2\pi} d\phi_k \sum_q \frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho *}(\mathbf{k})}{N_q^2} e^{ik_z(z-z')} \sum_{n=-\infty}^{\infty} (-i)^n H_n^{(2)}(k_{\rho q}\rho) \\ \qquad \times e^{-in(\phi-\phi_k)} \displaystyle\sum_{m=-\infty}^{\infty} i^m J_m(k_{\rho q}\rho') e^{im(\phi'-\phi_k)}, \qquad \rho \ge \rho'. \end{cases} \tag{36}$$

Here, we have employed the identity [4]

$$\int_0^\infty dk_\rho \frac{\bar{\mathbf{T}}[J_n(k_\rho\rho)J_n(k_\rho\rho')]}{(k_{\rho q}-k_\rho)N_q^2} = \frac{i\pi}{2N_q^2}\bar{\mathbf{T}}[H_n^{(2)}(k_{\rho q}\rho_>)J_n(k_{\rho q}\rho_<)], \tag{37}$$

where $\rho_> = \max(\rho,\rho')$, $\rho_< = \min(\rho,\rho')$, and $\bar{\mathbf{T}}$ stands for a dyadic operator, having the property of $\bar{\mathbf{T}}(k_\rho) = -\bar{\mathbf{T}}(-k_\rho)$.

Substituting the explicit expression of $\mathbf{\Psi}_q^\rho(\mathbf{k})$ in (36) and properly grouping the terms involving the integral with respect to the ϕ_k variable, the Green dyadics in the composite chiral-ferrite medium can be represented in terms of the cylindrical vector wave functions:

$$\begin{aligned} \bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}') = {} & \frac{i}{16\pi}\int_{-\infty}^{\infty} dk_z \sum_{q=1}^{2}\frac{1}{N_q^2}\sum_{n=-\infty}^{\infty}(-1)^n \\ & \times[a_q^\rho(k_{\rho q},k_z)\mathbf{M}_n^{(\tau_1)}(k_{\rho q},k_z) + b_q^\rho(k_{\rho q},k_z)\mathbf{N}_n^{(\tau_1)}(k_{\rho q},k_z) \\ & + c_q^\rho(k_{\rho q},k_z)\mathbf{L}_n^{(\tau_1)}(k_{\rho q},k_z)][a_q^{\rho'}(k_{\rho q},k_z)\mathbf{M}_{-n}^{(\tau_2)'}(k_{\rho q'},-k_z) \\ & + b_q^{\rho'}(k_{\rho q},k_z)\mathbf{N}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z) + c_q^{\rho'}(k_{\rho q},k_z)\mathbf{L}_{-n}^{(\tau_2)'}(k_{\rho q'},-k_z)]; \end{aligned} \tag{38}$$

$\bar{\mathbf{G}}_{\rm ee}(\mathbf{r},\mathbf{r}')$ can be obtained from $\bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}')$ with the replacement of a_q^ρ, b_q^ρ, c_q^ρ, $a_q^{\rho'}$, $b_q^{\rho'}$, $c_q^{\rho'}$ by d_q^ρ, e_q^ρ, f_q^ρ, $d_q^{\rho'}$, $e_q^{\rho'}$, $f_q^{\rho'}$ respectively; $\bar{\mathbf{G}}_{\rm em}(\mathbf{r},\mathbf{r}')$ can be derived from $\bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}')$ with the separate substitution of a_q^ρ, b_q^ρ, c_q^ρ by d_q^ρ, e_q^ρ, f_q^ρ; $\bar{\mathbf{G}}_{\rm me}(\mathbf{r},\mathbf{r}')$ can be obtained from $\bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}')$ with the replacement of $a_q^{\rho'}$, $b_q^{\rho'}$, $c_q^{\rho'}$ by $d_q^{\rho'}$, $e_q^{\rho'}$, $f_q^{\rho'}$, respectively. Here, $\tau_1 = 1$, $\tau_2 = 4$, for $\rho \le \rho'$ and $\tau_1 = 4$, $\tau_2 = 1$ for $\rho \ge \rho'$. The expansion coefficients used here can be straightforwardly obtained from equations (25–27) and (33–35), with the substitution of $\sigma = \rho$ and $k_\rho = k_{\rho q}$.

In equation (38), $k_{\rho 3}$ and $k_{\rho 4}$ are not included in the summation since $k_{\rho 3} = -k_{\rho 1}$, $k_{\rho 4} = -k_{\rho 2}$ and these symmetric roots are automatically taken into account as the spectral azimuthal angle ϕ_k spans from 0 to 2π.

It should be pointed out that the Green dyadics represented in the forms of the eigenfunction expansion, as given in this subsection, can be verified by comparing their special forms with the counterparts of reciprocal chiral medium [25] and isotropic medium [4] Moreover, they can be used to construct the Green dyadics of a cylindrically multilayered structure consisting of the composite chiral-ferrite media, by employing the method of scattering superposition [4,25] and appropriate electromagnetic boundary conditions.

The resulting equations in this subsection indicate that the electromagnetic waves in an unbounded composite chiral-ferrite medium are transversely outgoing for $\rho \geq \rho'$ and transversely standing for $\rho \leq \rho'$ This physical property of the electromagnetic waves is similar to that of a dielectric leaky antenna with infinitely long circular cylindrical structure, positioned in an unbounded isotropic medium.

From the present formulations, it is easily seen that $\bar{\mathbf{G}}_{RS}(\mathbf{r},\mathbf{r}') \not\equiv \bar{\mathbf{G}}^T_{RS}(\mathbf{r},\mathbf{r}')$ with or $R(S) = e$ or m. as can also be directly obtained from the reciprocal theorem [26] In addition, it can be straightforward to derive the mathematical relationship among these Green dyadics, which could also be obtained from the definition of the Green dyadics (18) and the source-incorporated Maxwell's equations (2):

$$\bar{\mathbf{G}}_{\rm em}(\mathbf{r},\mathbf{r}') = -\frac{i}{\omega\varepsilon}[(\nabla \times \bar{\mathbf{I}} + \omega\xi_c\bar{\mathbf{I}}) \cdot \bar{\mathbf{G}}_{\rm mm}(\mathbf{r},\mathbf{r}')], \tag{39}$$

$$\bar{\mathbf{G}}_{\rm me}(\mathbf{r},\mathbf{r}') = \frac{i}{\omega}\bar{\boldsymbol{\mu}}^{-1} \cdot [(\nabla \times \bar{\mathbf{I}} + \omega\xi_c\bar{\mathbf{I}}) \cdot \bar{\mathbf{G}}_{\rm ee}(\mathbf{r},\mathbf{r}')]. \tag{40}$$

The electromagnetic fields associated with the exciting sources can be obtained from (18), by substituting either set of the above-presented Green dyadics. From the present formulations, it can be seen that the solutions of the source-incorporated Maxwell's equations for a homogeneous composite chiral-ferrite medium are composed of two eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves (**M** and **N** represent two transverse waves) and a longitudinal wave. The chirality effects of the composite chiral-ferrite medium is manageable by introducing a controllable gyromagnetic parameter g to manage the wavenumbers of the eigenwaves propagating in this medium.

The essential idea of the method employed here, which is standard and straightforward, can be exploited to derive the eigenfunction expansion of the Green dyadics in a spherical coordinate system. However, since the wavenumbers of the eigenwaves are functions of the direction of these eigenwaves, simple compact forms of the field representations (corresponding to those of [9–12]) in the source-free composite chiral-ferrite media by the spherical vector wave functions cannot be obtained, and the solutions of the source-incorporated Maxwell's equations cannot be directly formulated in compact forms of the spherical vector wave functions, either. In a circular cylindrical coordinate system. however, it is seen from the present formulations that since the wavenumbers of the eigenwaves do not depend on the spectral azimuthal angle ϕ_k, the solutions of the source-incorporated Maxwell's equations in the composite chiral-ferrite medium can be represented in compact forms of the cylindrical vector wave functions.

4. Concluding Remarks

In the present contribution, eigenfunction expansion of the Green dyadics in an unbounded composite chiral-ferrite medium are developed in terms of the cylindrical vector wave functions, based on the concept of spectral eigenwaves. The analysis indicates that the solutions of the source-incorporated Maxwell's equations in a composite chiral-ferrite medium are composed of two eigenwaves travelling with different wavenumbers Each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. The Green dyadics of planarly and cylindrically multilayered structures consisting of composite chiral-ferrite media can be straightforwardly obtained by employing the method of scattering superposition and appropriate electromagnetic boundary conditions, respectively. The constraint condition of the present approach, which is standard and straightforward, is that the spectral longitudinal (and radial) wavenumbers do not depend on the spectral azimuthal angle ϕ_k. In spite of this constraint condition which makes the approach employed here only applicable to a limited class of materials, the present formulations can be used to analyze the physical phenomena of the source-incorporated electromagnetic boundary value problems involving unbounded or multilayered composite chiral-ferrite media. It is of interest to note that the cylindrical vector wave functions can be expanded as discrete sums of the spherical vector wave functions [32], therefore the present formulations could be extended to solve the problems of spherical structures. The present formulations can be theoretically verified by comparing their special forms with existing results corresponding to reciprocal chiral medium [24, 25]. In addition, the method employed here can be extended to derive the eigenfunction expansion of Green dyadics in other kinds of media, such as transversely isotropic elastic media [33], transversely isotropic piezoelectric solids [34], and transversely isotropic saturated porous media [35]. Although the present formulations are somewhat cumbersome which are inevitable due to the complexity of the material we have tried to tackle, they are important and useful in analyzing the (equivalently) source-incorporated electromagnetic phenomena of the composite chiral-ferrite media Even if there exist two operations in the present formulations which are over infinite domains, convergence of these operations has been numerically examined for the source-free problems [9–12]. For the source-incorporated problems, verification for the convergence of the operations is straightforward. Moreover, these two operations over infinite domains also exist for isotropic media [4] and reciprocal chiral media [24, 25], therefore various numerical and asymptotic methods [31] can be employed to simplify the computation in practical applications. It is believed that the present formulations provide fundamental basis to analyze the (equivalently) source-incorporated electromagnetic phenomena of the composite chiral-ferrite media. Applications of the present formulations in analyzing the electromagnetic scattering, propagation, resonance, and radiation phenomena relevant to the composite chiral-ferrite media are under investigation, and will be reported in the near future.

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